

Power Control Game with SINR-Pricing in Variable-Demand Wireless Data Networks

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Abstract—Game theory has been applied to model power control in wireless systems for years. Conventional power control games tend to consider unlimited backlogged user traffic. Different from the conventional methodology, this paper aims to investigate limited backlogged data traffic and construct a game-theoretic model tackling both one-shot and repeated power control problem in which variable user traffic demand needs to be taken into consideration. To improve the network performance in such situation, we devise a new SINR pricing scheme and propose an algorithm to calculate an optimal price. With this optimal price, we prove that the Nash equilibrium is Pareto efficient and max-min fair.

I. INTRODUCTION

Uplink power control is critical to CDMA systems where users compete for limited network resource and the actions of users make direct impact on one another. Such characteristic makes game theory a suitable base to model uplink power control problem. In contrast to the conventional unlimited data assumption, we investigate a power control problem in which user traffic demand varies. We use game theory and design a pricing scheme to prevent the waste of network resource, which leads to an efficient Nash equilibrium.

Extensive research has been done in power control problems. Koskie and Gajic, for instance, provide an overview of power control problems [1]. As Meshkati *et al.* mention [2], there are two kinds of utility function. Alpcan *et al.* construct a model based on Shannon capacity to show that the Nash Equilibrium is unique [3]. Altman *et al.* discuss two types of decision making: the centralized system and the decentralized system, the former corresponds to cellular network while the latter to ad hoc network [4]. Gunturi *et al.* propose a linear pricing scheme based on power level to show the system converges to a unique NE [5]. Goodman *et al.* propose a model considering transmission error [6]. A more efficient model adopting the linear pricing scheme is later proposed [7].

We aim to solve the power control problem by proposing a new pricing scheme that is a linear function of SINR. Since the purpose of pricing is to eliminate the waste of system resource, the criterion of pricing should be related to the usage of common resource. Therefore, the power level, which consumes user's own energy, is not an ideal criterion of pricing. On the contrary, since every user competes for throughput on which SINR has direct effect, a more ideal criterion could be based on SINR. In addition to the pricing scheme, unlike conventional power control games focusing on unlimited data traffic, we formulate a game in which the traffic demand is limited, as

this might be the case for the future generation high-speed wireless data networks.

II. SYSTEM MODEL

We begin this section with a one-period game. The set \mathcal{N} contains n users, each of which is endowed with the traffic demand M_i to upload to the base station, $i = 1, 2, \dots, n$. Each user cares only its own uploading traffic and adjusts its power level to maximize its throughput. The SINR of the user i is:

$$\gamma_i = \frac{p_i}{\sum_{j \neq i} p_j + N_0} \quad (1)$$

where $p_i \in [0, \bar{p}_i]$ refers to the power level of the user i and N_0 the noise. Channel gain is not included in this equation, since we consider the received power at the base station.

The valuation function v_i of the user i is Shannon Capacity $\delta \log(1 + \gamma_i)$, where δ is a constant transferring the unit of the valuation function to become identical to the unit of demand. Since the demand is limited, two conditions are concerned: 1) $\delta \log(1 + \gamma_i) \geq M_i$ implies the demand is met; 2) $\delta \log(1 + \gamma_i) < M_i$ implies the demand is not met. The valuation reaches its maximum when the demand is met. With the pricing scheme, the utility function of the user i is:

$$u_i = v_i - c\gamma_i = \min[M_i, \delta \log(1 + \gamma_i)] - c\gamma_i \quad (2)$$

The following proposition indicates that the price influences the willingness of user's participation in this game.

Proposition 1 *No user joins the system if $c \geq \delta$.*

It can be showed that the utility is negative when $c > \delta$ and is zero when $c = \delta$. Since the price cannot be negative, the price must fall within the range of $0 \leq c < \delta$.

Definition 1 *The required SINR γ_i^c is the SINR with which the user i exactly meets its demand.*

$$\gamma_i^c = e^{\frac{M_i}{\delta}} - 1$$

The definition is derived from $\delta \log(1 + \gamma_i^c) = M_i$. When one's SINR exceeds the required SINR, the user meets the demand. Before deriving the equilibrium, we first discuss the problem of finding the corresponding vector of power when the desired vector of SINR is given [1]. When such corresponding vector of power exists, the desired vector of SINR is called feasible. The problem is solved by Sampath *et al.* [8] who prove that the vector of SINR is feasible if the following feasibility condition holds.

$$\sum_{j=1}^n \frac{1}{\frac{1}{\gamma_j} + 1} \leq 1 - \frac{N_0}{\min_i [\bar{p}_i (\frac{1}{\gamma_i} + 1)]} \quad (3)$$

Thereinafter, a vector of SINR may not be feasible. However, we focus on the feasible vector of SINR throughout this paper and derive the price optimizing the system performance. Moreover, when the vector of SINR is feasible, the corresponding vector of power can be calculated by solving the simultaneous equations. Therefore, we consider SINR to be the strategy rather than power in the following sections.

The *optimal SINR* is defined as the SINR with which the user maximizes the utility. We show that a user has two possibilities of the optimal SINR as follows.

Lemma 1 *The user i has two possibilities of the optimal SINR, denoted by $\gamma_i^{*(1)}$ and $\gamma_i^{*(2)}$ respectively.*

$$\begin{cases} \gamma_i^{*(1)} = \gamma_i^c & , \gamma_i \geq e^{\frac{M_i}{\delta}} - 1 \\ \gamma_i^{*(2)} = \frac{\delta}{c} - 1 & , \gamma_i < e^{\frac{M_i}{\delta}} - 1 \end{cases} \quad (4)$$

Proof:

- Case I: $\gamma_i \geq e^{\frac{M_i}{\delta}} - 1$

With the equivalent condition $\delta \log(1 + \gamma_i) \geq M_i$, the utility of the user who can meet its demand is $u_i = M_i - c\gamma_i$. We then derive $\frac{\partial u_i}{\partial \gamma_i} = -c < 0$, which implies once the user meets its demand, the utility would only decrease. Thus, the user has to meet its demand exactly. Formally, $\delta \log(1 + \gamma_i^{*(1)}) = M_i$. Equivalently, $\gamma_i^{*(1)} = e^{\frac{M_i}{\delta}} - 1$.

- Case II: $\gamma_i < e^{\frac{M_i}{\delta}} - 1$

With the equivalent condition $\delta \log(1 + \gamma_i) < M_i$, the utility of the user who fails to meet its demand is $u_i = \delta \log(1 + \gamma_i) - c\gamma_i$. To prove that the utility function is concave and has its maximum, we have $\frac{\partial^2 u_i}{\partial \gamma_i^2} = \frac{-1}{(1 + \gamma_i)^2} \leq 0$ and $\frac{\partial u_i}{\partial \gamma_i} = \frac{\delta}{1 + \gamma_i^{*(2)}} \cdot \frac{\partial \gamma_i^{*(2)}}{\partial p_i} - c \frac{\partial \gamma_i^{*(2)}}{\partial p_i} = 0$. Therefore, the maximum is $\gamma_i^{*(2)} = \frac{\delta}{c} - 1$. ■

Either of the two possibilities corresponds to one mutual exclusive domain of the strategy space. Besides, Lemma 1 rules out other possibilities, leaving only two. In next theorem, we prove the user has only one possibility of the optimal SINR.

Theorem 1 *The optimal SINR of the user i is:*

$$\gamma_i^* = \begin{cases} \gamma_i^c & , M_i \leq \delta \log \frac{\delta}{c} \\ \frac{\delta}{c} - 1 & , M_i > \delta \log \frac{\delta}{c} \end{cases} \quad (5)$$

The proof is similar to the proof of Lemma 1. Theorem 1 shows that the optimal SINR is determined by the demand of the user. In addition, since the boundary of condition is a function of δ and c , the base station can decide the number of user meeting the demand by choosing a proper price.

We denote the boundary of demand in Theorem 1 by the *threshold demand* $M_{th} = \delta \log \frac{\delta}{c}$ and denote the required SINR of M_{th} by the *threshold SINR* $\gamma_{th} = e^{\frac{M_{th}}{\delta}} - 1 = \frac{\delta}{c} - 1$. Then there are two kinds of users: 1) the users endowed with demand $M_i \leq M_{th}$ are contained in Ω^C ; 2) the users endowed with demand $M_i > M_{th}$ are contained in Ω^N .

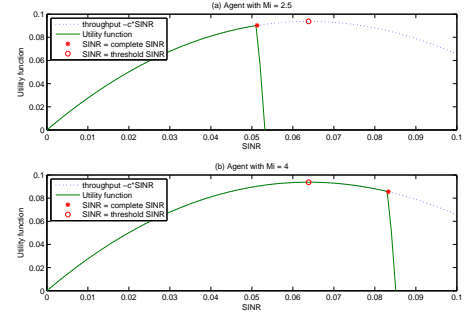


Fig. 1. Utility of the user in Ω^C and Ω^N

Therefore, the users belonging to Ω^C can meet the demand, while the users belonging to Ω^N cannot. Note that $\Omega^C \cup \Omega^N = \mathcal{N}$ and $\Omega^C \cap \Omega^N = \emptyset$.

For instance, one user endowed with $M = 2.5$ belongs to Ω^C and the other endowed with $M = 4$ belongs to Ω^N . Fig.1(a) and Fig.1(b) demonstrate the utility functions of these two users respectively. For the user belonging to Ω^C , it has already met the demand before reaching $\gamma^{*(2)}$. Thus it chooses $\gamma^{*(1)}$ to be the optimal SINR. On the other hand, the user belonging to Ω^N chooses $\gamma^{*(2)}$, since the user would obtain lower utility when choosing $\gamma^{*(1)}$.

We find the optimal SINR is uniquely determined by the demand, and not affected by other users. Thus, the equilibrium is uniquely determined by the optimal SINRs of users, and then by the demand distribution of users.

Definition 2 *Let S_i be the strategy set of the user i , where $S_i = [0, \bar{p}_i]$. Denote $S = S_1 \times S_2 \dots \times S_n$ as the strategy profile, and S_{-i} the strategy profile except S_i . The Nash Equilibrium (NE) with the strategy vector (p_i^*, p_{-i}^*) is that:*

$$u_i(p_i^*, p_{-i}^*) \geq u_i(p_i, p_{-i}^*) \quad \forall p_i \in S_i, p_{-i}^* \in S_{-i}, \forall i \quad (6)$$

Theorem 2 *If the vector of optimal SINR is feasible, there is a unique NE where all users adopt the optimal SINR.*

$$\gamma_i^* = \begin{cases} \gamma_i^c & , i \in \Omega^C \\ \gamma_{th} & , i \in \Omega^N \end{cases}$$

The existence of NE is directly based on the feasibility of the vector of optimal SINR. Likewise, NE is unique since the optimal SINR is unique for all users.

III. OPTIMAL PRICE

The notation *optimality* is used twice throughout this paper. One represents the best reaction, denoted by the *optimal SINR*. The other represents the price optimizing the system performance in NE, denoted by the *optimal price*. We derive the optimal price in this section.

If the price decreases, the optimal SINRs of all users belonging to Ω^N increase, and the total throughput rises along with it, which improves the system performance. However, when some of the optimal SINRs raise, the feasibility condition (3) may be violated (i.e., the vector of optimal SINR becomes

infeasible), which is an unwanted result. On the other hand, if the price increases, the optimal SINRs of all users belonging to Ω^N decrease, and so does the total throughput, which worsens the system performance. Therefore, the optimal price should be a price that optimizes the total throughput without violating the feasibility condition (3).

Definition 3 *The optimal price c^* is the lowest price such that the vector of the optimal SINR is feasible. Formally,*

$$c^* = \min\{c \mid \gamma^* \text{ is feasible}\}, \text{ where } \gamma^* = (\gamma_1^*, \gamma_2^*, \dots, \gamma_n^*)$$

We have $0 \leq c^* < \delta$ directly from Proposition 1. Moreover, if the optimal price is adopted, the NE has two properties: Pareto efficiency and max-min fairness.

Definition 4 *Let S_i be the strategy set of the user i , where $S_i = [0, \bar{p}_i]$. Denote $S = S_1 \times S_2 \dots \times S_n$ as the strategy profile, and denote S_{-i} as the strategy profile except S_i . The Pareto efficient strategy set (p_i^*, p_{-i}^*) is that:*

$$v_i(p_i^*, p_{-i}^*) \geq v_i(p_i, p_{-i}) \forall p_i \in S_i, \forall p_{-i} \in S_{-i}, \forall i \quad (7)$$

In Pareto efficient NE, any increasing of valuation of one user must be in the expense of the decrease of valuation of some other users. There is no way that users can increase their valuation simultaneously.

Theorem 3 *If the optimal price is adopted, the Nash Equilibrium is Pareto efficient.*

Proof: There are two cases. If all users belong to Ω^C , then the valuation of all users reach the maximum. Hence, the NE is Pareto efficient. On the other hand, if some users belong to Ω^N , for the user i whose $p_i = \bar{p}$, then the only way to increase its valuation is to decrease the power of other users, which lowers the valuation of certain users. ■

Definition 5: *A strategy set S is max-min fair if and only if for all users, any increase of the valuation must be at a cost of decrease of some originally smaller valuation. Formally, for any other S' such that $v_i(S') > v_i(S)$, there must exist at least one user j such that $v_i(S) \geq v_j(S)$ and $v_j(S') < v_j(S)$.*

In max-min fair equilibrium, the user originally with lower valuation has better chances to increase its valuation without violating the max-min fairness. On the contrary, the user with higher valuation tends to violate the max-min fairness.

Theorem 4 *When the optimal price is adopted, the Nash Equilibrium is max-min fair.*

Proof: From Theorem 3, it is shown that any increase of a user's valuation decreases the valuation of some other users. For the users belonging to Ω^C , there is no way to increase their valuation. For the users belonging to Ω^N , since all the other users have at most the equal valuation as them, any increase of their valuation certainly lowers some of originally smaller valuation. ■

Therefore, the base station can optimize the total throughput by adopting the optimal price. In the next section, we design an algorithm to calculate the optimal price.

IV. ALGORITHM DESIGN

The algorithm is denoted by Binary Convergent Pricing Algorithm (BCPA). To calculate the optimal price, the base

station needs the information about the traffic demand M_i and the upper bound of power \bar{p}_i of all users.

Simply speaking, BCPA has five steps:

- 1) Sort users according to the demand in an ascending order, and group the users who share the same demand.
- 2) Store the minimal value of \bar{p}_i in every group.
- 3) Set the price c to $\delta/2$. (i.e., the middle of $0 \leq c < \delta$.)
- 4) If the vector of optimal SINR is feasible, decrease the price c . Otherwise, increase price c .
- 5) Do step 4 recursively by binary approximation. Terminate when the vector of optimal SINR is about to become infeasible, and then return the price.

BCPA is used in the numerical result. To generalize the result, we derive the two-period model in the next section.

V. REPEATED STAGES

In this section, we give the insight of the multi-period model by deriving the last two-period model. Consider the finite-period model with the period $T-1$ and T , where the period T is the last period before the game ends. The length of the period $T-1$ is the same as that of the period T . The valuation v_i^{T-1} is the sum of the valuations during the two periods. We denote the arriving demand before the period $T-1$ and before the period T by M_{T-1} and M_T respectively. The demand at the period $T-1$ will be accumulated if it is not met.

At the period T , since all users know there is no next period, the equilibrium is the same as Theorem 1. Thus, the optimal price at the period T is the optimal price in the one-period game, denoted by c_T . In this section, we derive the optimal SINRs at the period $T-1$. Note that the optimal price at the period T is affected by the strategy at the period $T-1$, because part of the demand M_{T-1} will be accumulated to the period T when the demand is not met. The remaining definitions are the same as those of the one-period game.

The strategy in two-period game is determined at the period $T-1$. The user can manipulate the strategy at the period $T-1$ to make itself meet the demand at the period T or not.

- $i \in \Omega_T^C$ (i.e., the demand at the period T is met) iff

$$\delta \log(1 + \gamma_i^T) \geq M_T + \max[0, M_{T-1} - \delta \log(1 + \gamma_i^{T-1})] \quad (8)$$
- $i \in \Omega_T^N$ (i.e., the demand at the period T is not met) iff

$$\delta \log(1 + \gamma_i^T) < M_T + \max[0, M_{T-1} - \delta \log(1 + \gamma_i^{T-1})] \quad (9)$$

Users belonging to Ω_T^C or Ω_T^N have different valuation functions. The valuation of the user i at the period $T-1$, denoted by v_i^{T-1} , is that:

- If $i \in \Omega_T^C$ then

$$v_i^{T-1} = M_{T-1} + M_T \quad (10)$$

- If $i \in \Omega_T^N$ then

$$v_i^{T-1} = \min(M_{T-1}, \delta \log(1 + \gamma_i^{T-1})) + \delta \log \frac{\delta}{c_T} \quad (11)$$

The user meets the demand at the period T implies that both M_{T-1} and M_T are met, since the remaining demand at the period $T-1$ would be accumulated. However, if the user

fails to meet the demand at the period T . The reason could be either the accumulation of M_{T-1} plus M_T or merely M_T is too much to be met.

The reaction at the period T is determined by the strategy at the period $T-1$. The following equation shows their relation.

$$\begin{aligned} & \delta \log(1 + \gamma_i^T) \\ &= \min(\delta \log \frac{\delta}{c_T}, M_T + \max[0, M_{T-1} - \delta \log(1 + \gamma_i^{T-1})]) \end{aligned} \quad (12)$$

The second term in the operator min is the total demand at the period T . If M_{T-1} is not met, the remaining demand will be added to the demand at the period T . The first term is the maximum of the second term, as we mention in Theorem 1.

The objective of the user i is to maximize the utility.

$$\begin{aligned} & \max_{\gamma_i^{T-1}} u_i^{T-1} \\ \equiv & \max_{\gamma_i^{T-1}} v_i^{T-1} - c_T \gamma_i^T - c_{T-1} \gamma_i^{T-1} \end{aligned}$$

subject to (10), (11), and (12)

We solve this optimization problem by dividing the domain of strategy space into four parts.

- When $\delta \log(1 + \gamma_i^{T-1}) \geq M_{T-1}$

1) By (12), we have

$$\delta \log(1 + \gamma_i^T) = \min(\delta \log \frac{\delta}{c_T}, M_T) \quad (13)$$

Then we have $\delta \log \frac{\delta}{c_T} - M_T \leq 0$.

$$i \in \Omega_T^N \text{ iff } \begin{cases} \delta \log(1 + \gamma_i^{T-1}) \geq M_{T-1} \\ \delta \log(1 + \gamma_i^T) - M_T < 0 \end{cases} \quad (14)$$

$$i \in \Omega_T^C \text{ iff } \begin{cases} \delta \log(1 + \gamma_i^{T-1}) \geq M_{T-1} \\ \delta \log(1 + \gamma_i^T) - M_T = 0 \end{cases} \quad (15)$$

- When $\delta \log(1 + \gamma_i^T) < M_{T-1}$

1) There are two cases under this condition.

– When

$$\begin{cases} \delta \log(1 + \gamma_i^{T-1}) < M_{T-1} \\ \delta \log(1 + \gamma_i^{T-1}) < M_{T-1} + M_T - \delta \log \frac{\delta}{c_T} \end{cases} \quad (16)$$

- 1) By (12), $\delta \log(1 + \gamma_i^T) = \delta \log \frac{\delta}{c_T}$.
- 2) $\delta \log(1 + \gamma_i^T) - (M_{T-1} + M_T - \delta \log(1 + \gamma_i^{T-1})) < \delta \log \frac{\delta}{c_T} - \delta \log \frac{\delta}{c_T} = 0$.
- 3) From (9), user i belongs to Ω_T^N .

– When

$$\begin{cases} \delta \log(1 + \gamma_i^{T-1}) < M_{T-1} \\ \delta \log(1 + \gamma_i^{T-1}) \geq M_{T-1} + M_T - \delta \log \frac{\delta}{c_T} \end{cases} \quad (17)$$

- 1) $\delta \log(1 + \gamma_i^T) = M_T + M_{T-1} - \delta \log(1 + \gamma_i^{T-1})$
- 2) From (8), user i belongs to Ω_T^C .

Proposition 2 Only the users belonging to Ω_T^C can affect c_T .

To prove it, we consider the case in which the demand M_{T-1} is not met. For the users belonging to Ω_T^N , no matter how much the remaining demand is, they still cannot meet the demand at the period T . On the contrary, for the users belonging to Ω_T^C , they can adjust their power to meet its new

demand. The original vector of the optimal SINR becomes infeasible, so c_T must be recalculated. The other case in which M_{T-1} is met is similar to the former one. We can then solve the optimization problem with this proposition.

1) Case I: From (15)

$$\begin{cases} \delta \log(1 + \gamma_i^{T-1}) \geq M_{T-1} \\ \delta \log(1 + \gamma_i^T) = M_T \end{cases}$$

User i belongs to Ω_T^C . The utility function is:

$$u^{T-1} = M_{T-1} + M_T - c_T \gamma_i^T - c_{T-1} \gamma_i^{T-1}$$

From Proposition 3, the user can affect c_T . However, since all demand is met, the user does not change c_T .

The best reaction is:

$$B_1 : \begin{cases} \gamma_i^{T-1} = e^{\frac{M_{T-1}}{\delta}} - 1 \\ \gamma_i^T = e^{\frac{M_T}{\delta}} - 1 \end{cases}$$

2) Case II: From (14)

$$\begin{cases} \delta \log(1 + \gamma_i^{T-1}) \geq M_{T-1} \\ \delta \log(1 + \gamma_i^T) < M_T \end{cases}$$

User i belongs to Ω_T^N . The utility function is:

$$u^{T-1} = M_{T-1} + \delta \log \frac{\delta}{c_T} - c_T \gamma_i^T - c_{T-1} \gamma_i^{T-1}$$

The user cannot affect the price c_T . From (13), we have $\delta \log(1 + \gamma_i^T) = \delta \log \frac{\delta}{c_T}$. The best reaction is:

$$B_2 : \begin{cases} \gamma_i^{T-1} = e^{\frac{M_{T-1}}{\delta}} - 1 \\ \gamma_i^T = \frac{\delta}{c_T} - 1 \end{cases}$$

3) Case III: From (16)

$$\begin{cases} \delta \log(1 + \gamma_i^{T-1}) < M_{T-1} \\ \delta \log(1 + \gamma_i^{T-1}) < M_T + M_{T-1} - \frac{\delta}{c_T} \end{cases}$$

User i belongs to Ω_T^N . The utility function is:

$$u^{T-1} = \delta \log(1 + \gamma_i^{T-1}) + \delta \log \frac{\delta}{c_T} - c_T \gamma_i^T - c_{T-1} \gamma_i^{T-1}$$

The user cannot affect the price c_T . We have that $\delta \log(1 + \gamma_i^T) = \delta \log \frac{\delta}{c_T}$. Hence, the best reaction is similar to the one in the one-period game.

$$B_3 : \begin{cases} \gamma_i^{T-1} = \begin{cases} e^{\frac{M_{T-1}}{c_{T-1}}} - 1, & M_{T-1} \leq \delta \log \frac{\delta}{c_{T-1}} \\ \frac{\delta}{c_{T-1}} - 1, & M_{T-1} > \delta \log \frac{\delta}{c_{T-1}} \end{cases} \\ \gamma_i^T = \frac{\delta}{c_T} - 1 \end{cases}$$

4) Case IV: From (17)

$$\begin{cases} \delta \log(1 + \gamma_i^{T-1}) < M_{T-1} \\ \delta \log(1 + \gamma_i^T) \geq M_T + M_{T-1} - \frac{\delta}{c_T} \end{cases}$$

User i belongs to Ω_T^C . The utility function is:

$$u^{T-1} = M_{T-1} + M_T - c_T \gamma_i^T - c_{T-1} \gamma_i^{T-1}$$

The user can affect the price c_T . Also, $\delta \log(1 + \gamma_i^T) = M_T + M_{T-1} - \delta \log(1 + \gamma_i^{T-1})$. Since both c_T and c_{T-1} are the function of γ_i^{T-1} , the best reaction exists.

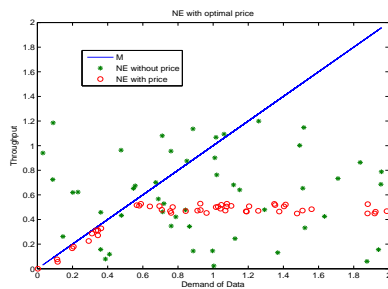


Fig. 2. The effect of the pricing scheme

TABLE I
SIMULATION SETTING

	\bar{p}	M_1	M_2	Total Demand
User 1	10	0.21	0.21	0.42
User 2	10	0.21	0.37	0.58
User 3	15	0.37	0.21	0.58
User 4	20	0.37	0.37	0.74

Environmental setting: $\delta = 1$, $N_0 = 1$

The strategy space in the two-period model has four possibilities of optimal SINRs. The result is similar to Lemma 1. If we bring in proper assumptions, the NE then becomes similar to that of the one-period game. Moreover, c_T is equal to or greater than c_{T-1} in the equilibrium; otherwise, some users can obtain a higher utility by reserving some demand from the period $T - 1$ to the period T .

VI. NUMERICAL RESULT

We show the influence of the proposed pricing scheme in Fig.2. The x-axis represents the demand of users, and the y-axis the throughput those users obtain in NE. Without pricing, some users waste the resource. However, when the optimal price is adopted, it can avoid such problem.

To observe the difference between NE in the two-period model and in the one-period model, we devise two schemes with two periods. The first scheme includes two one-period games denoted by Game 1 and Game 2. The second scheme includes one two-period game. In the latter scheme, the demand at the period 1 could be accumulated. The parameters of both schemes are presented in Table I.

The result of the first scheme is in Table II. As Theorem 1 states, those users who fail to meet the demand share the same throughput. On the other hand, the result of the second scheme is presented in Table III. User 1, 2, 3, and 4 respectively belongs to Case I, II, IV, and III in the previous section. At the period 1, both User 3 and User 4 fail to meet the demand, but, at the period 2, User 3 can meet M_2 plus the remaining demand of M_1 , while User 4 cannot, because the demand of User 3 does not exceed the threshold demand so User 3 can increase its power. Moreover, at the period 2, since User 3 chooses higher power level to meet the total demand, both User 2 and User 4 suffer from lower throughput because of the increase of power of User 3. The number of users meeting the demand increases from one in the first scheme to two in the

TABLE II
FIRST SCHEME (ONE-PERIOD GAMES)

	Game 1 throughput	Game 2 throughput	Summed throughput
User 1	0.21 (100%)	0.21 (100%)	0.42 (100%)
User 2	0.21 (100%)	0.35 (95%)	0.56 (97%)
User 3	0.357 (96%)	0.21 (100%)	0.567 (98%)
User 4	0.357 (96%)	0.35 (95%)	0.707 (96%)

Price in Game 1: $c = 0.6995$

Price in Game 2: $c = 0.7043$

TABLE III
SECOND SCHEME (TWO-PERIOD GAME)

	Period 1 throughput	Period 2 throughput	Total throughput
User 1	0.21 (100%)	0.21 (100%)	0.42 (100%)
User 2	0.21 (100%)	0.345 (93%)	0.555 (96%)
User 3	0.359 (97%)	0.011(3%)+0.21 (100%)	0.58 (100%)
User 4	0.357 (96%)	0.345 (93%)	0.702 (95%)

Prices in Period 1: $c_1 = 0.6995$

Prices in Period 2: $c_2 = 0.7082$

second scheme. We conclude that the pricing scheme increases the number of user whose demand is fully met.

VII. CONCLUSION

We investigate a SINR-based pricing scheme with limit traffic demand. If the optimal price is adopted, the Nash Equilibrium is Pareto efficient and max-min fair. To obtain this optimal price, we design an algorithm called BCPA. We also show that the multiple-period model has the similar equilibrium as one in the one-period model. The best reaction of the user is also determined by its traffic demand, and there exists a unique Nash Equilibrium.

ACKNOWLEDGMENT

This work was supported in part by the Ministry of Economic Affairs in Taiwan under Grant 98-EC-17-A-02-S2-0127, and by the National Science Council of Taiwan under Grant NSC-98-2219-E-002-019-.

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